

# On low energy quantum gravity induced effects on the propagation of light.

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## Abstract.

Present models describing the interaction of quantum Maxwell and gravitational fields predict a breakdown of Lorentz invariance and a non standard dispersion relation in the semiclassical approximation. Comparison with observational data however, does not support their predictions. In this work we introduce a different set of ab initio assumptions in the canonical approach, namely that the homogeneous Maxwell equations are valid in the semiclassical approximation, and find that the resulting field equations are Lorentz invariant in the semiclassical limit.

We also include a phenomenological analysis of possible effects on the propagation of light, and their dependence on energy, in a cosmological context.

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## 1. Introduction

Present models describing the interaction of quantum Maxwell and gravitational fields predict a breakdown of Lorentz invariance. In the semiclassical approximation, the common feature in these models is a non standard dispersion relation which shows that the spacetime behaves as a media with a frequency dependent index of refraction [1, 2, 3, 4]. Another consequence is the selection of a preferred reference frame, namely, the one at rest with respect to the media. In geometrical terms this can be realized by the introduction of a preferred timelike vector field  $t^a$  that serves as a universal time.

It is instructive to analyze in pure phenomenological terms the effect of a semiclassical gravity state on the propagation of electromagnetic radiation. The effective interaction is realized by adding to the standard Maxwell equations extra terms with minimal coupling between the Maxwell field and the above mentioned timelike vector field. Although the interaction contains parity breaking and parity preserving terms, it is perhaps surprising that the only possible measurable effect arising from our approach comes from the parity breaking interaction and it is this effect, precisely, what is predicted by the leading models on the canonical approach [2, 3, 4]. The main prediction of these models is a dispersion relation that depends on the helicity and the energy of the radiation field. Our analysis also seems to rule out wave propagation that is parity invariant, and in particular recent results by John Ellis, et. al [5].

Note that if the dispersion relation depends on the helicity then a linearly polarized wave packet with a continuum spectra will rotate its polarization direction as it travels through space. Since the rotation depends on the energy, the wave will become totally depolarized after traveling a suitable optical path. For light coming from a cosmological source, the observation of linear polarization can be used to set a severe bound on the phenomenological coupling constant of the Gambini-Pullin model [6] and, as we will show in this work, essentially rule out the Sahlmann-Thiemann construction.

At this point one is tempted to ask whether or not Lorentz invariance is necessarily broken by quantum gravity. Since observational evidence seems to support Lorentz invariance, we reexamine the canonical models looking for assumptions introduced in their construction that, although natural at first sight, might not be true in the final theory. Our hope is that by suitably changing them we may preserve Lorentz invariance.

In particular, in what follows we replace their ab-initio assumption of the electric field and potential being conjugate variables by a different ansatz, namely, that the quantum Maxwell field is the curvature of the Maxwell connection. This natural assumption has the expected consequence that the resulting field equations are Lorentz invariant. Although the calculation is done at a linear level, it is very easy to extend the results to any order in the perturbation parameter. We thus conjecture that Lorentz invariance will still hold in the quantum interaction between gravitons and photons.

The work is organized as follows. In Section 2 we present a phenomenological approach to the propagation equations showing that if Lorentz invariance is broken, at most there will be a gravity induced rotation of the polarization vector. We also

show that the available observational data essentially rules out the Sahlmann-Thiemann model.

In Section 3 we assume that the Maxwell field is the curvature of a connection, namely,  $F = dA$ . Since this statement is independent of the existence of a quantum gravitational field we conjecture that it is valid for the full theory. Using the energy density of the electromagnetic field and this new assumption, we obtain different conjugate variables and thus different field equations for the Maxwell field from those obtained in previous canonical models.

Finally, in the conclusions we summarize the main results obtained in this work. Is Lorentz invariance broken when light propagates on a quantum space-time? We address this question and add our own bias on the subject.

## 2. Phenomenological equations for the propagation of light

The construction of equations for the propagation of light under the influence of effects arising from the quantum nature of gravity requires explicit assumptions about the form that we expect the low energy regime of these effects will take. In the particular case of cosmological applications, we would expect that the quantum expectation values should be in consonance with the properties of the classical metric  $g_{ab}$ , defining the classical geometry. For the standard models, this is characterized by a timelike vector field  $t^a$ , whose integral lines are the world lines of comoving observers with 4-velocity  $u^a$ . We take this as implying that the expectation values defining the (local) low energy limit should be tensor functions only of  $u^a$ ,  $g_{ab}$  and a scalar function of  $t$ , where  $t$  is an affine parameter (“time”) for the integral lines of  $t^a$ .

In view of some recent theoretical results [1] we take the Maxwell tensor  $F_{ab}$  as the fundamental physical quantity describing the electromagnetic field. Its components in a local Lorentz frame are  $F_{0i} = E_i$  and  $F_{ij} = \epsilon_{ijk}B_k$ , where  $E_i$  and  $B_k$  are, respectively, the components of the electric and magnetic field vectors, and where  $\epsilon_{ijk}$  is the Levi-Civita symbol. We, therefore, do not assume the existence of a vector potential  $A_a$ . In the absence of quantum gravity effects, the propagation of light is governed by the equations for the electromagnetic field  $F_{ab}$ , which may be written as

$$\nabla^a F^*_{ab} = 0, \tag{1}$$

$$\nabla^a F_{ab} = 4\pi J_b, \tag{2}$$

where  $F^*_{ab}$  is the dual of  $F_{ab}$ ,  $J_a$  is the electric current, and we recall that the vanishing of the right hand side of (1) corresponds to the absence of magnetic type currents and monopoles.

The modification of these equations that we envision is the addition of terms on the right hand sides of (1) and (2), corresponding to the presence of different “effective currents” possibly resulting from quantum gravity effects and to which the electromagnetic field gets coupled. We shall restrict to terms that are linear in the

electromagnetic field, assuming that non linear effects are of higher order and may be neglected in this approximation. Our phenomenological equations take the form,

$$\begin{aligned}\nabla^a F^*_{ab} = & \psi_1 t^a F_{ab} + \psi_2 t^a F^*_{ab} \\ & + \psi_3 t^a t^c \nabla_c F_{ab} + \psi_4 t^a t^c \nabla_c F^*_{ab} \\ & + \psi_5 t^a t^c t^d \nabla_c [\nabla_d F_{ab}] + \psi_6 t^a t^c t^d \nabla_c [\nabla_d F^*_{ab}] \\ & + \psi_7 t^a g^{cd} \nabla_c [\nabla_d F_{ab}] + \psi_8 t^a g^{cd} \nabla_c [\nabla_d F^*_{ab}],\end{aligned}\tag{3}$$

$$\begin{aligned}\nabla^a F_{ab} = & 4\pi J_b + \chi_1 t^a F_{ab} + \chi_2 t^a F^*_{ab} \\ & + \chi_3 t^a t^c \nabla_c F_{ab} + \chi_4 t^a t^c \nabla_c F^*_{ab} \\ & + \chi_5 t^a t^c t^d \nabla_c [\nabla_d F_{ab}] + \chi_6 t^a t^c t^d \nabla_c [\nabla_d F^*_{ab}] \\ & + \chi_7 t^a g^{cd} \nabla_c [\nabla_d F_{ab}] + \chi_8 t^a g^{cd} \nabla_c [\nabla_d F^*_{ab}].\end{aligned}\tag{4}$$

In these equations we are assuming local couplings that lead to expressions in  $F_{ab}$  and its derivatives, and we have included only terms up to second derivatives. Notice that, from our assumption of a cosmological metric we have,

$$\begin{aligned}t^a \nabla_a t^b &= 0, \\ \nabla^a t^b - \nabla^b t^a &= 0.\end{aligned}\tag{5}$$

Similarly, from simple physical arguments, we expect the coefficients  $\chi_i$  and  $\psi_i$  to depend only on the scale parameter (“radius of the Universe”) of the metric, in such a way that they vanish when the Planck length  $\ell_P$  is taken equal to zero. Note that even if some or all of the coefficients  $\chi_i$  and  $\psi_i$  are non zero, on observational grounds they must be small. Thus, we may consider the effect of each term separately, as any cross effects would be of higher order. This analysis is carried out in the next subsection.

### 2.1. Local Lorentz frame analysis

The equations are more easily analyzed in a local Lorentz frame, adapted to the symmetry of the metric. Namely, if the local coordinates are  $(x, y, z, t)$ , we have  $t^a = (0, 0, 0, 1)$ . As a first approximation we also neglect curvature effects, and equate covariant to ordinary partial derivatives. Moreover, we assume  $J^a = 0$ , corresponding to “vacuum” propagation, and take all  $\psi_i$ , and  $\chi_i$  as constants.

Using the standard forms for  $F_{ab}$  and  $F^*_{ab}$ , we immediately find

$$\nabla \cdot \vec{E} = 0,\tag{6}$$

$$\nabla \cdot \vec{B} = 0,\tag{7}$$

and

$$\nabla \times \vec{E} + \partial_t \vec{B} = \chi_1 \vec{E} + \psi_1 \vec{B} + \chi_2 \partial_t \vec{E} + \psi_2 \partial_t \vec{B}$$

$$+ \chi_3 \partial_t^2 \vec{E} + \psi_3 \partial_t^2 \vec{B} + \chi_4 \nabla^2 \vec{E} + \psi_4 \nabla^2 \vec{B}, \quad (8)$$

$$\begin{aligned} \nabla \times \vec{B} - \partial_t \vec{E} = & \chi_5 \vec{B} + \psi_5 \vec{E} + \chi_6 \partial_t \vec{B} + \psi_6 \partial_t \vec{E} \\ & + \chi_7 \partial_t^2 \vec{B} + \psi_7 \partial_t^2 \vec{E} + \chi_8 \nabla^2 \vec{B} + \psi_8 \nabla^2 \vec{E}. \end{aligned} \quad (9)$$

We remark that in (8) and (9) we have regrouped some terms from (3) and (4), and renamed some constants. Notice that the factors of  $\chi_i$  violate parity conservation. It can also be seen from (8) that the constants  $\psi_1, \dots, \psi_4$ , and  $\chi_1, \dots, \chi_4$  should vanish if one assumes absence of magnetic currents. In this latter case  $F_{ab}$  admits a vector potential  $A_a$ . This situation will be analyzed in the next Section.

## 2.2. Plane waves

We look now for plane wave solutions traveling along the z-axis. In this case, on account of (6), we should have,

$$\begin{aligned} \vec{E} &= \text{Re} \{ (E_x \hat{e}_x + E_y \hat{e}_y) \exp(i(\omega t - kz)) \}, \\ \vec{B} &= \text{Re} \{ (B_x \hat{e}_x + B_y \hat{e}_y) \exp(i(\omega t - kz)) \}. \end{aligned} \quad (10)$$

Replacing (10) in (8) and (9) we find the general form for the dispersion relation  $k = k(\omega, \chi_i, \psi_i)$ . In view of the smallness of  $\chi_i$  and  $\psi_i$ , this may be expanded up to linear order in this coefficients, but it will be convenient to keep higher order terms. If we consider separately terms by their order of derivatives, and their parity conserving or violating character we find, in the parity conserving cases:

$$\begin{aligned} k &= \sqrt{\omega^2 + 2i\omega(\psi_1 + \psi_5) - 4\psi_1\psi_5} \\ &\simeq \omega + i(\psi_1 + \psi_5), \end{aligned} \quad (11)$$

$$\begin{aligned} k &= \omega \sqrt{(1 - 2(\psi_2 + \psi_6) + 4\psi_2\psi_6)} \\ &\simeq \omega[1 - (\psi_2 + \psi_6)], \end{aligned} \quad (12)$$

$$\begin{aligned} k &= \omega \sqrt{[1 - 2i\omega(\psi_3 + \psi_7) - 4\omega^2\psi_3\psi_7]} \\ &\simeq \omega[1 - i\omega(\psi_3 + \psi_7)], \end{aligned} \quad (13)$$

$$\begin{aligned} k &= \omega \left[ \sqrt{1 - k^2(\psi_4 - \psi_8)^2 + ik(\psi_4 + \psi_8)} \right]^{-1} \\ &\simeq \omega[1 - i\omega(\psi_4 + \psi_8)], \end{aligned} \quad (14)$$

and for the parity violating cases:

$$\begin{aligned} k &= \sqrt{\omega^2 \pm 2k(\chi_1 - \chi_5) + 4\chi_1\chi_5} \\ &\simeq \omega \pm (\chi_1 - \chi_5), \end{aligned} \quad (15)$$

$$\begin{aligned} k &= \omega \left[ \sqrt{1 - (\chi_2 + \chi_6)^2 \pm i(\chi_2 - \chi_6)} \right] \\ &\simeq \omega[1 \pm i(\chi_2 - \chi_6)], \end{aligned} \quad (16)$$

$$k = \omega \left[ \sqrt{1 + \omega^2(\chi_3 + \chi_7)^2 \pm \omega(\chi_3 - \chi_7)} \right]$$

$$\simeq \omega[1 \pm \omega(\chi_3 - \chi_7)], \quad (17)$$

$$k = \omega \left[ 1 \pm 2k(\chi_4 - \chi_8) - 4k^2\chi_4\chi_8 \right]^{-1/2} \\ \simeq \omega[1 \pm \omega(\chi_4 - \chi_8)]. \quad (18)$$

If we use these results to obtain the corresponding amplitudes, we find that in the parity conserving cases there is a single mode of propagation, and the polarization of a plane polarized wave is preserved in time, while in the parity violating cases there are two modes, one corresponding to right, and the other the left circular polarization. Possible observational effects for these cases have been discussed in [6].

Considering now particular cases, we see that (11) corresponds to a frequency independent amplification or attenuation (depending on the sign of  $\psi_1 + \psi_5$ ) of the wave amplitude with time, with no effects on the polarization. The effect would be absent for  $\psi_1 = -\psi_5$ , but this would still leave a term quadratic in  $\psi_1$ , that would behave as a tachyonic mass added to the photon. The important question here is the order of magnitude of these couplings. In the absence of a theory we may only conjecture that we would expect these to be of the order of the corresponding dimensional quantities that characterize the model. In our case these are the Planck length  $\ell_P$ , and time  $t_P$  and possibly, for instance, the radius (or scale)  $a(T)$ , and age  $T$  (or the Hubble constant  $H = \dot{a}/a$ ) of the Universe. Then, since  $\psi_1$  and  $\psi_5$  should have dimension  $[\text{length}]^{-1}$ , one would expect these quantities to be of the order of  $t_P H^2$ , which, after multiplication by the time to travel through cosmological distances, is too small to have any phenomenological relevance.

In the case (12) we have a frequency independent change in the velocity of the waves, which would be larger or smaller than that of light, depending on the sign of  $\psi_2 + \psi_6$ . Since  $\psi_2$  and  $\psi_6$  are dimensionless, an estimate for them could be  $t_P H$ , which is again too small for observable consequences.

The case (14) is conceptually similar to (11), leading again to amplification or attenuation of the waves. However, in this case  $\psi_4$  and  $\psi_8$  have dimension of  $[\text{length}]$ . Then, if we take them to be of the order of  $\ell_P$ , the relevant quantity would be of order  $\ell_P L / \lambda^2$ , where  $\lambda$  is wavelength of the wave and  $L$  of the order of the distance to cosmological sources. Perhaps surprisingly, if we assume  $\lambda = 10^{-5} \text{cm}$ , (for visible light), and  $L = 10^9 \text{ly}$ , we find already  $\ell_P L / \lambda^2 \simeq 10^3$ . This would imply that visible light would not reach us but, in fact, not only visible light but also  $\gamma$ -rays are observed from cosmological sources. This implies that either  $\psi_4$  and  $\psi_8$  are much smaller than this scale or  $\psi_4 \simeq -\psi_8$ . If we take  $\psi_4 = -\psi_8$ , we find  $k \simeq \omega - 2\psi_8^2 \omega^3$ . This implies a group velocity  $v_g = 1 - 6\psi_8^2 \omega^2$  which, even for high energy  $\gamma$ -rays and cosmological times, gives an effect that is too small for observable consequences.

The case (13) is similar to (14), but here we might question if it is acceptable to have second order time derivatives on the right hand side of the equation, or we should disregard this possibility.

Similar considerations regarding orders of magnitude and observability hold for the parity violating cases (15), (16), (17), and (18). The last case, with  $\chi_4 = -\chi_8$ , was

obtained in [2]. We notice however that the available observational data indicates that  $\chi_4 \simeq 10^{-4}$ [6], much smaller than the expected value of order one.

It is worth mentioning that the same observational data can be used to test the validity of a recent result that also predicts a rotation of the polarization vector [4]. Denoting by  $D$  the cosmological distance to the source of the incoming radiation, by  $\theta_{GP} = \chi \ell_P k^2 D$  and  $\theta_{ST} = \ell_P^{2\alpha} L^{1-2\alpha} k^2 D$  the rotation angle of the polarization direction obtained by Gambini-Pullin and Sahlmann-Thiemann respectively, and comparing the uncertainty of these angles with the observational data, one obtains the following restriction for the constants  $\alpha$  and  $L$ ,

$$\left(\frac{\ell_P}{L}\right)^{2\alpha-1} \simeq \chi \simeq 10^{-4}.$$

However, these constants must satisfy the following inequalities  $0 < \alpha < \frac{1}{2}$  and  $L \gg \ell_P$ [4], which clearly violate the above equation. This would suggest that the kinematical states used by the authors to derive their equations cannot be considered as semiclassical states.

### 3. The interaction between the gravitational and electromagnetic fields

In this section we first review the standard classical Lagrangian and Hamiltonian formulation of coupled Maxwell and gravitational fields. This review is then used to argue that, even when we promote the fields to quantum operators, there will always be relations that remain valid for the quantum theory.

The maxwell field on a curved spacetime is given by an exact 2-form

$$F = dA,$$

with  $A$  the Maxwell connection 1-form. Given a local coordinate system with timelike and spacelike coordinate vectors  $e_o^a$  and  $e_i^a$  respectively, we define the electric and magnetic fields as  $E_i = F_{io}$  and  $B_i = \epsilon_{ijk} F_{jk}$ , where  $\epsilon_{ijk}$  is the Levi-Civita symbol. With these definitions (and assuming that  $A_o = 0$  since we are only concerned with propagating waves) the relations between the fields and the potential can be written as

$$E_i = -\partial_t A_i, \tag{19}$$

and

$$B_i = \epsilon_{ijk} \partial_j A_k. \tag{20}$$

Note that these relations do not depend on any metric and are just the coordinate components of the above equation.

Assume now that we give any Lagrangian density  $\mathcal{L} = \mathcal{L}(F, g)$  describing the coupling between the Maxwell and gravitational field. By definition, the Maxwell field will satisfy

$$dF = 0,$$

since it is not part of the Euler-Lagrange equations of motion. The coupling between the metric and electromagnetic fields will of course depend on the detailed form of the Lagrangian density. In particular, for

$$\mathcal{L}_\gamma = -\frac{1}{2}F \wedge *F \quad (21)$$

we obtain the standard field equations

$$d^*F = 0,$$

but we will allow for more general types of couplings that might arise from the semiclassical approximation when we promote the classical fields to quantum operators. To construct a Hamiltonian formulation we first define the canonical momentum conjugated to  $A_i$

$$\pi^i = \frac{\partial \mathcal{L}}{\partial(\partial_t A_i)}$$

and then invert this relation to obtain

$$\partial_t A_i = G_i(\pi^j, A_k). \quad (22)$$

The Legendre transformation  $\mathcal{H} = \pi^i G_i - \mathcal{L}$  then gives the desired Hamiltonian density. Note that, by construction, the hamilton equation of motion for  $A_i$  will be (22). Note also that, regardless of the form of  $G_i$ ,  $E_i$  must satisfy equation (19). Thus, we expect that for a non-standard interaction Hamiltonian,  $-E_i$  will not be the conjugate momentum to  $A_i$ .

If we now promote the classical fields to quantum operators  $(\hat{A}, \hat{F}, \hat{g})$  we might expect several changes in the quantum hamiltonian but the basic definitions should remain valid. For example,

$$\hat{F} = d\hat{A},$$

says that  $\hat{F}$  is still the curvature of  $\hat{A}$ . Thus, the equation

$$d\hat{F} = 0,$$

should hold as an identity for  $\hat{A}$ . Moreover, the above relation does not depend on  $\hat{g}$ , therefore even if we must regularize the metric operator it should remain unchanged. Even if we take expectation values of this relation and assume the state is a direct product of a coherent state for the Maxwell field and a “semiclassical state” for the gravitational field one should reobtain

$$F_{class} = dA_{class}, \quad dF_{class} = 0,$$

at least at a semiclassical approximation. Thus, the plan is as follows:

- (i) We start by imposing  $\hat{F} = d\hat{A}$ .



- (ii) We then take the regularized energy density of the electromagnetic field written in terms of  $E_i$  and  $B_i$  and write a modified relation between  $E_i$  and  $\pi^i$  so that, via the hamilton equations, the relation

$$E_i = -\partial_t A_i$$

holds order by order in a perturbation expansion.

- (iii) With the desired relation  $E_i = -G_i$  we obtain the second equation of motion for  $\pi^i$  and then rewrite the final result as a modified set of equations for the Maxwell fields.

### 3.1. The phenomenological Hamiltonian and Lagrangian densities

Following the Gambini-Pullin approach[2], the phenomenological interaction Hamiltonian density (obtained by taking expectation values of the regularized quantum hamiltonian with coherent states for the Maxwell field and “weave states” for the gravitational field) is given by,

$$\mathcal{H}_{\mathcal{EB}} = \frac{1}{2} (\vec{E}^2 + \vec{B}^2) + \chi l_p (\vec{E} \cdot \vec{\nabla} \times \vec{E} + \vec{B} \cdot \vec{\nabla} \times \vec{B}). \quad (23)$$

where  $\chi$  is a phenomenological coupling constant that destroys parity invariance.

The relationship between the fields and the conjugate variables  $(\vec{A}, \vec{\pi})$  is, by assumption, given by

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (24)$$

$$\vec{E} = -\vec{\pi} + \chi l_p \vec{F}[\vec{\pi}], \quad (25)$$

where we have adopted the vectorial notation for ease of writing and where  $\vec{F}[\vec{\pi}]$  is a function of  $\vec{\pi}$  and its derivatives. If we now insert these relations in the above Hamiltonian and impose that  $\vec{E} = -\partial_t \vec{A}$  (which must be true by definition), the functional  $\vec{F}$  is determined in a unique way via the Hamilton equation for  $\vec{A}$ , leading to

$$\vec{E} = -\vec{\pi} + 2\chi l_p \vec{\nabla} \times \vec{\pi}. \quad (26)$$

This allows us to obtain the second equation of motion for  $\vec{\pi}$ . Hamilton equations for the conjugate variables then read

$$\partial_t \vec{A} = \vec{\pi} - 2\chi l_p \vec{\nabla} \times \vec{\pi}, \quad (27)$$

$$\partial_t \vec{\pi} = -\vec{\nabla} \times \vec{\nabla} \times (\vec{A} + 2\chi l_p \vec{\nabla} \times \vec{A}). \quad (28)$$

Combining these equations and assuming the Coulomb gauge we obtain

$$(\partial_t^2 - \nabla^2) \vec{A} = \mathcal{O}((\chi l_p)^2). \quad (29)$$

Alternatively, we can derive Maxwell’s equations for the electric and magnetic fields yielding

$$\nabla \times \vec{B} - \partial_t \vec{E} = \mathcal{O}((\chi l_p)^2), \quad (30)$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = \mathcal{O}\left((\chi l_p)^2\right), \quad (31)$$

$$\nabla \cdot \vec{E} = \mathcal{O}\left((\chi l_p)^2\right), \quad (32)$$

$$\nabla \cdot \vec{B} = 0. \quad (33)$$

**Remarks:**

- Note that our field equations are Lorentz invariant and have a standard dispersion relation.
- Starting from the same eq. (23), Gambini and Pullin[2] assumed a different relation between the electric field and the momentum, namely  $\vec{E} = -\vec{\pi}$ . This assumption leads to the following equations of motion for the conjugate pair,

$$\partial_t \vec{A} = -\left(\vec{E} + 2\chi l_p \nabla \times \vec{E}\right), \quad (34)$$

$$\partial_t \vec{E} = \nabla \times \left(\vec{B} + 2\chi l_p \nabla \times \vec{B}\right). \quad (35)$$

In terms of the fields, the equations can be rewritten as

$$\nabla \times \vec{B} - \partial_t \vec{E} = 2\chi l_p \nabla^2 \vec{B}, \quad (36)$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = -2\chi l_p \nabla^2 \vec{E}, \quad (37)$$

which, unlike the ones we obtained, are not Lorentz invariant. It is important to notice that this breakdown of invariance follows from the assumption that  $-\vec{E}$  and  $\vec{A}$  are canonical variables. Although this is the case for the classical theory, it might not remain valid when we promote the fields to quantum operators. On the other hand, the quantum version of  $\vec{E} = -\partial_t \vec{A}$  should remain valid since this is just a consequence of  $F = dA$ . We therefore conjecture that the homogeneous Maxwell equations, contained in  $F = dA$ , should remain unchanged for the full theory.

- It is worth mentioning that using a completely different quantization procedure Ellis et. al. obtain a generalized set of Maxwell equations that keep the homogeneous part unchanged[5].

It is also possible to derive, up to linear order in  $\chi$ , the Lagrangian density associated to the Hamiltonian (23), via the Legendre transformation  $\mathcal{L} = \pi^i \partial_t A_i - \mathcal{H}$ . If we invert (26) we get

$$\vec{\pi} = -\vec{E} - 2\chi l_p \vec{\nabla} \times \vec{E}, \quad (38)$$

which leads to the following Lagrangian density

$$\mathcal{L}_{\mathcal{EB}} = \frac{1}{2} \left( \vec{E}^2 - \vec{B}^2 \right) + \chi l_p \left( \vec{E} \cdot \vec{\nabla} \times \vec{E} - \vec{B} \cdot \vec{\nabla} \times \vec{B} \right). \quad (39)$$

The Euler-Lagrange equation of motion for the potential can be written in terms of the fields as:

$$\nabla \times \vec{B} - \partial_t \vec{E} = -2\chi l_p \nabla \times \left( \nabla \times \vec{B} - \partial_t \vec{E} \right) + \mathcal{O}\left((\chi l_p)^2\right), \quad (40)$$

Note also that in principle the above equations could give more solutions than plane waves since they are second order PDE's. Thus, there seems to be more solutions to the

above equations than Hamiltons equations (30), since the ones obtained directly from the Hamiltonian do not contain terms linear in  $\chi$ . The extra terms appearing in (40) arise when we invert the relation between  $\vec{E}$  and  $\vec{\pi}$ . We can see that both equations contain the same solutions, in the linear approximation, by the following consideration. Let  $\vec{C} = \vec{\nabla} \times \vec{B} - \partial_t \vec{E}$ . The solution to Hamilton equations correspond to  $\vec{C} = 0$ . We now search for non trivial solutions to

$$2\chi l_p \vec{\nabla} \times \vec{C} + \vec{C} = 0, \quad (41)$$

Note that if  $\vec{C}$  satisfies (41), then it is also a solution of

$$\vec{C} + (2\chi l_p)^2 \nabla^2 \vec{C} = 0, \quad (42)$$

and this implies that solutions of (41) have a “monochromatic” (spatial) Fourier spectrum with  $k = (2\chi l_p)^{-1}$ . Such contributions with wavelength of the order of the Planck length must be absent if the low energy approximation (characteristic lengths much larger than  $l_p$ ) is to be consistent.

Assuming this cut off is implemented, the only solution consistent with our approximation is  $\vec{\nabla} \times \vec{B} - \partial_t \vec{E} = 0$ .

Using a somewhat different approach, L. Urrutia[8] developed a Lagrangian formulation that leads to equations (36, 37). Starting from (21) and defining  $\vec{E}$  by Eq. (34) and  $\vec{B}$  as the curl of  $\vec{A}$ , Urrutia obtains a Lagrangian density for  $\vec{A}$  such that the corresponding Euler-Lagrange equations yield (36, 37). This Lagrangian, however, is a non-local functional of  $\vec{A}$ . The fact that a Lorentz violating dispersion relation  $\omega_{\pm} = \sqrt{k^2 \mp 4\chi l_p k^3} \simeq |k|(1 \mp 2\chi l_p |k|)$  is found for the set of equations (36, 37) is consistent with a recent result advocating that non-local Lagrangians can generate non-Lorentz-invariant dispersion laws[9]. The non-local Lagrangian arose, however, from an assumed non-local relation between  $\vec{E}$  and  $\vec{A}$  which will not be true if one follows our assumption.

Unlike the approach of Urrutia[8] the Lagrangian density that yields eqs.(30,31) is a local functional of the potential. This result is in agreement with recent work of J. Bros and H. Epstein [10] which states that microcausality and energy positivity in all frames imply Lorentz invariance of dispersion laws. (Within the domain of our assumption the energy spectrum will be positive definite since the Hamiltonian can be written as  $\mathcal{H}_{\mathcal{EB}} = \frac{1}{2} (\vec{E} + \chi l_p \vec{\nabla} \times \vec{E})^2 + \frac{1}{2} (\vec{B} + \chi l_p \vec{\nabla} \times \vec{B})^2$  plus terms of order  $\chi^2$ ).

It would appear that the issue of whether or not Lorentz invariance is broken depends on the specific relation between the fields and the potential. The local Lagrangian constructed from the potential and its derivatives yields Lorentz invariant dispersion laws, whereas the non-local Lagrangian breaks the invariance. Following our assumption that  $\hat{F} = d\hat{A}$  we conjecture that the quantum fields will be local functionals of  $\hat{A}$ .

In principle, if the Hamiltonian were given as a perturbation expansion in  $\chi$ , it would be possible to introduce higher order corrections to the relationship between the electric field and conjugate momentum, eq. (26), so that Faraday’s law is preserved at each order in that expansion.

#### 4. Final comments and conclusions

We first summarize the main results of this work. In Section 2 we concentrate on the phenomenological field equations for Maxwell fields that could arise from the interaction with the gravitational field in the semiclassical approximation. We show that, up to linear order, there is just one acceptable way to obtain a non-standard dispersion relation, namely, field equations with parity violating coupling constants[2].

Is the universe living in a state of definite parity? Voting for the affirmative, the cosmological nature and detailed temporal structure of gamma ray bursts could serve as a tool to observe the predicted deviations from the standard dispersion relations. As was shown on a previous work[6] the phenomenological constant is rather small and future observations could be useful to decide if indeed it is non-vanishing. However, using the available data we can essentially rule out the Sahlmann-Thiemann construction.

In view of the present observational data for cosmological sources and the ambiguities of the available models for the interaction of Maxwell fields with a semiclassical space-time it is fair to ask ourselves whether or not we should still have faith in Lorentz invariance. In Section 3 we show that a very natural assumption leads to Lorentz invariant field equations. Our conclusion is that we have no reason to believe that a quantum theory of gravity would change the invariance and thus conjecture that Lorentz invariance will still hold in the quantum interaction between gravitons and photons.

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